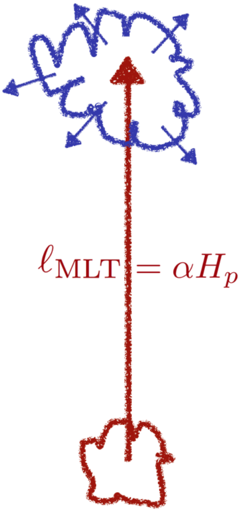
# Entropy-calibrated modelling of solar type stars

Louis Manchon, F. Spada, M. Deal,
A. Serenelli, M.-J. Goupil, H.-G. Ludwig, L. Gizon

Journées de la SF2A, 7-10 juin 2022

MAX PLANCK INSTITUTE
FOR SOLAR SYSTEM RESEARCH

How do we model convection in 1D stellar evolution codes?

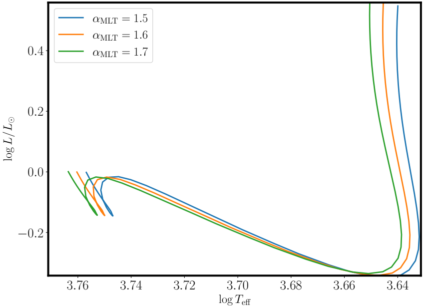



- Convection : extremely complex ⇒ *ad hoc* theory : MLT, CGM, ... ⇝ free parameter.
- MLT : Hot gas parcel rises to a height $\ell \propto \alpha_{\mathrm{MLT}} H_p$. $\alpha_{\mathrm{MLT}}$ controls the convective flux.
- How do we choose $\alpha$?
  - From calibration;
  - Compute grid of models with different $\alpha$
  - Set to solar value;

Is there a more physical way to choose it?
Should $\alpha$ stays fixed along evolution?

## Can $\alpha$ be linked to other quantities?

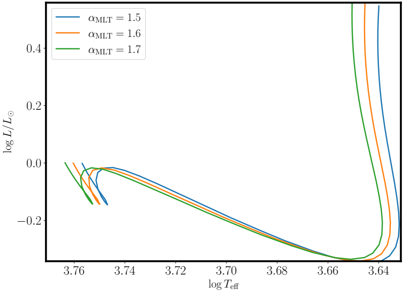
$\alpha$ controls the stellar radius $R$.



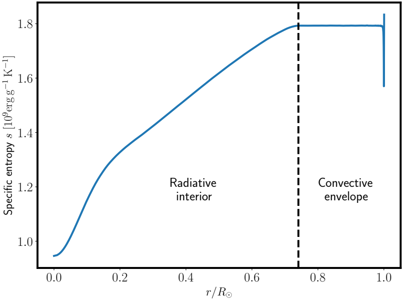

$$L = 4\pi R^2 \sigma T_{\mathrm{eff}}^4$$

# Can $\alpha$ be linked to other quantities?

$\alpha$ controls the stellar radius $R$.




But $R$ is also controlled by $s_{\rm ad}$, the entropy of the adiabat.




E.g., in a polytropic, completely convective model,

$$R \propto \exp\left(\frac{\gamma - 1}{3\gamma - 4}\,\frac{\mu s_{\rm ad}}{N_{\rm A} k_{\rm B}}\right)$$

**Then $\alpha$ and $s_{\rm ad}$ are linked.**

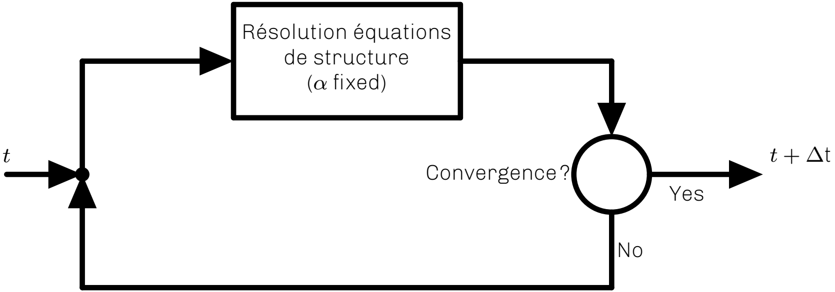
## How do we know what $s_{\rm ad}$ a star should have ?

• From precise 3D modelling of convective upper layers (stagger, CO5BOLD, ...). $s_{\rm ad}$ **is an input of the models.**
• Using grids of 3D atmospheres : prescription for $s_{\rm ad}$ as a function of $T_{\rm eff}, \log g, Z$.
• So far, three prescriptions :

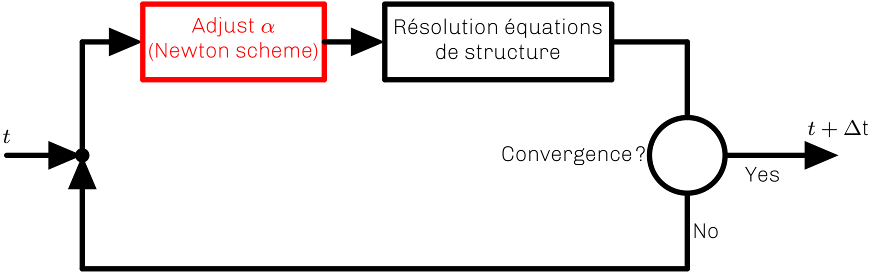
- Ludwig+99 : Based on 2D atm. models at fixed metallicity and with a chemical composition close to GS98 (proto-Sun).
- Magic+13 : Based on 3D atm. models (STAGGER grid). $[{\rm Fe/H}] \in [-4.0; +0.5]$. Chem. composition : $\simeq$ AGS09 (pres. Sun).
- Tanner+16 : Same as Magic+13 but with different mathematical form.

$\Rightarrow$ we can determine $s_{\rm ad}$ knowing $T_{\rm eff}, \log g$ and $Z$.
How do we relate $\alpha$ to $s_{\rm ad}$ ?

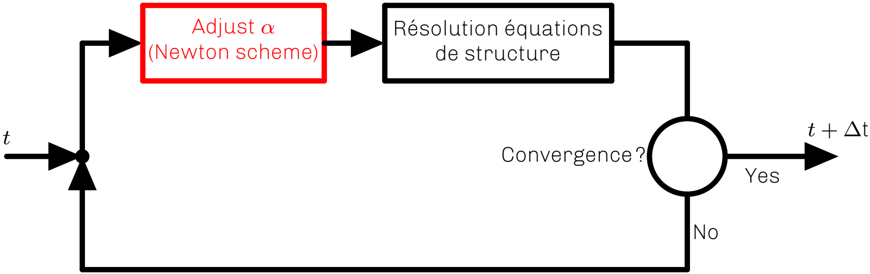
**In a traditional stellar evolution code** (e.g CESTAM ; Morel+95, Lebreton+08, Marques+13) :



**Entropy-calibration, general idea (Spada+2018,2019,2021) :**
The goal is to adjust $\alpha$ along evolution so that $s_{\rm ad}$ in 1D models matches $s_{\rm MHD}$ obtained from prescriptions.



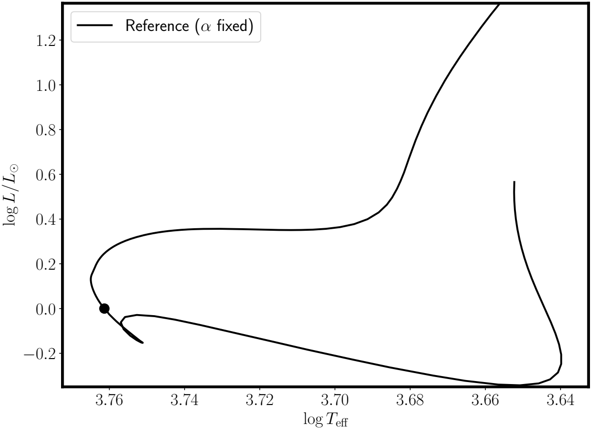
**Entropy-calibration, general idea (Spada+2018,2019,2021) :**
The goal is to adjust $\alpha$ along evolution so that $s_{\rm ad}$ in 1D models matches $s_{\rm MHD}$ obtained from prescriptions.



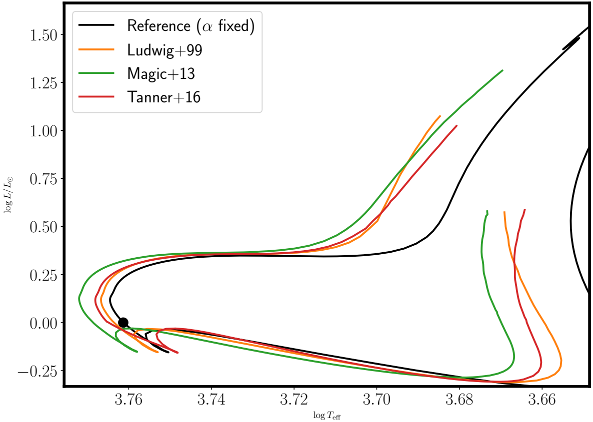

New implementation in CESTAM.
Why redo the work of Spada+ ?

- Different code (YREC $\rightarrow$ CESTAM). Different way of computing convective envelope ;
- Lot of care should be taken when using entropy prescriptions.

Gravitational settling, GS98.
Tuning of $Y_0$ and $\alpha_{\mathrm{MLT}}(\simeq 1.81)$ to obtain $T_{\mathrm{eff},\odot}$, $L_\odot$.



# A first result




⇒ Large discrepancies ($\Delta T_{\rm eff} > 100$K).

It's different. Is it better?

# Prescriptions should be corrected

- Entropy is defined up to a constant. EoS tables used in 2D and 3D MHD models and 1D evolution models are not the same.
  $\Rightarrow$ Addition of an offset $\mathrm{d}s$, computed using a reference model (Spada+2018,2019).
- The entropy varies with the chemical composition :

$$s \propto \frac{1}{\mu} \ln(\ldots) \tag{1}$$

  The mean molecular weight $\mu$ is different in MHD models and in your 1D model.
  $\Rightarrow$ Multiplicative factor $f_\mu = \mu_{\mathrm{RHD}}/\mu_{\mathrm{1D}}$ (Spada+2021).

Final corrected form :

$$s_{\mathrm{MHD}} \rightsquigarrow s_{\mathrm{MHD}} f_\mu + \mathrm{d}s \tag{2}$$

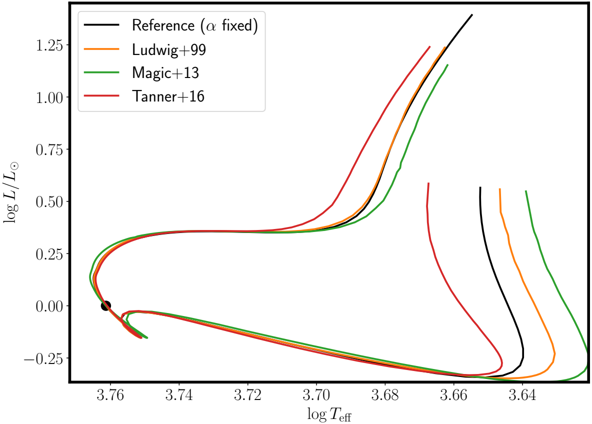
# Prescriptions should be corrected

But also, prescription's coefficients from original paper should be used.

- Ludwig+99 Based on 2D models $\rightarrow$ less accurate adiabatic entropies
- Magic+13 & Tanner+16 : original paper used entropies at the bottom of the simulate box instead of $s_{\mathrm{ad}}$

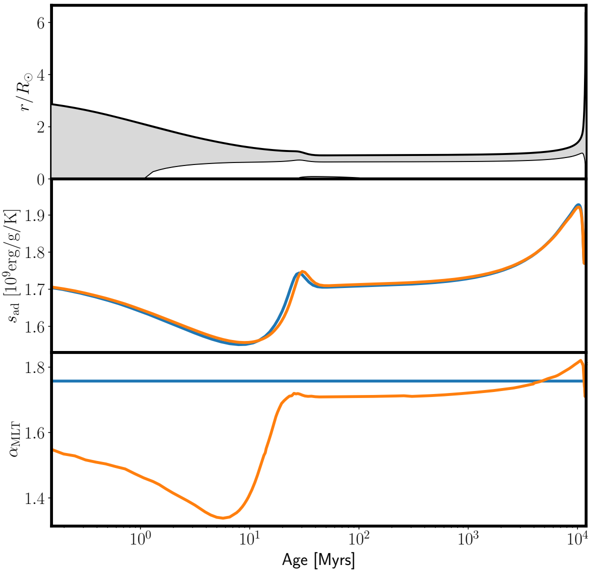
Using data from the CIFIST grid, we recalibrated all the parameters for the different prescriptions.

# Better results




What is the cause of discrepancies during PMS and RGB?

# $\alpha_{\rm MLT}$ evolution




$s \propto \ln T^{3/2}/\rho$
Virial th. : $T \propto R^{-1}$,
$\rho \propto R^{-3}$

- PMS : contraction phase.

$\Rightarrow s \searrow$

- RGB : expansion phase

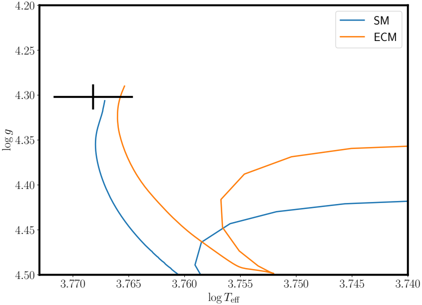
$R \nearrow \Rightarrow s \nearrow$

# 16Cyg A

- Observables (16 CygA; Karovicova+2021) :

| $\log g$ | $T_{\rm eff}$ | $L/L_\odot$ | [Fe/H] |
|---|---|---|---|
| $4.302 \pm 0.014$ | $5864 \pm 48$ | $1.511 \pm 0.0043$ | $0.15 \pm 0.05$ |

- Calibration through Levenberg-Marquardt algorithm (OSM ; R. Samadi).
- Physics : AGS09, MLT, gravitational settling.
- Standard model (SM). Adjustable parameters : Age, $M$, $\alpha_{\rm MLT}$ (fixed), $Y_0$. Targets : $\log g$, $T_{\rm eff}$, $L/L_\odot$ and [Fe/H].
- Entropy-calibrated model (ECM). Adjustable parameters : Age, $M$, $Y_0$. Targets : $\log g$, $T_{\rm eff}$ and [Fe/H].




| | $\log g$ | $T_{\rm eff}$ | $L/L_\odot$ | [Fe/H] | |
|---|---|---|---|---|---|
| Obs. | $4.302 \pm 0.014$ | $5864 \pm 48$ | $1.511 \pm 0.0043$ | $0.15 \pm 0.05$ | |
| SM | 4.306 | 5850 | 1.511 | 0.19 | |
| ECM | 4.290 | 5826 | 1.600 | 0.18 | |
| | Age | $M$ | $\alpha_{\rm MLT}$ | $Y_0$ | $R/R_\odot$ |
| SM | 7617 | $1.06 \pm 0.006$ | $2.02 \pm 0.098$ | $0.273 \pm 0.004$ | 1.20 |
| ECM | $6293 \pm 939$ | $1.10 \pm 0.03$ | Varying | $0.267 \pm .002$ | 1.24 |

PLATO expected accuracies. Age :10%; Mass : 15%, Radius : 2%.

# Conclusions

- Numerical scheme is robust and we recover results obtained by F. Spada with YREC.
- Sorted out the different prescriptions and improved them through corrections (see Manchon+, in prep for more details).
- Large impact for PLATO accuracy of model-dependent parameters.
- Changes PMS and RGB location of Solar type stars.
- Calibration independent of physics (contrary to prescription of $\alpha$).

Now :

- More detailed tests on benchmark stars (seismic,...).
- What impact it has on depth of CZ ? Could have an impact on transport processes.

**Thank you !**

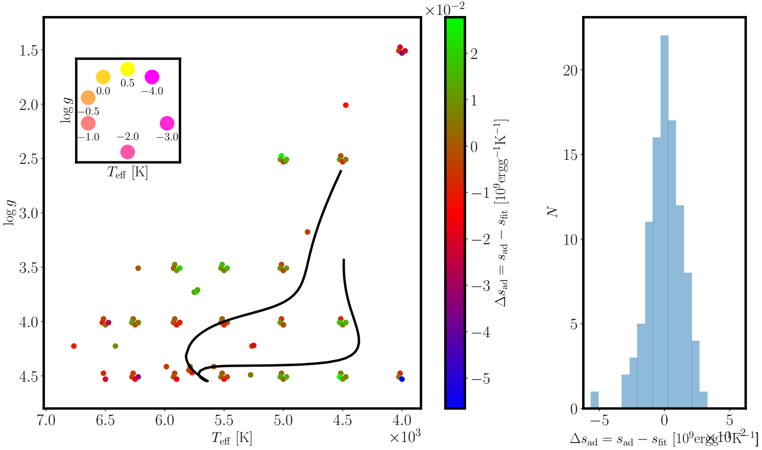
## Can $\alpha$ be linked to other quantities?

$\alpha$ controls the stellar radius $R$.
But $R$ is also controlled by $s_{\mathrm{ad}}$, the entropy of the adiabat.
In a polytropic, completely convective model, $p = K\rho^{\gamma}$ and $s = \frac{N_{\mathrm{A}} k_{\mathrm{B}}}{\mu} \ln K$.
With $K \propto M^{2-\gamma} R^{3\gamma-4}$, we have,

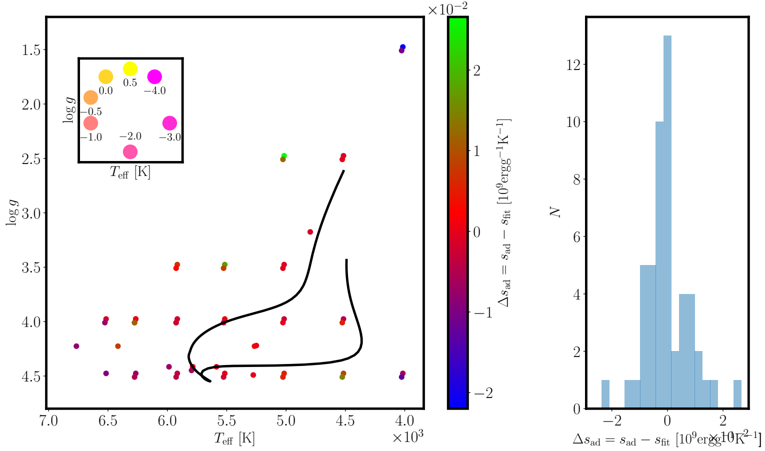
$$R \propto \exp\left(\frac{\gamma - 1}{3\gamma - 4} \frac{\mu s_{\mathrm{ad}}}{N_{\mathrm{A}} k_{\mathrm{B}}}\right) \tag{3}$$

with $\gamma$ the adiabatic exponent.

Error between Magic+13 prescription, with coefficients calibrated on the CIFIST grid



Error between Magic+13 prescription, with coefficients calibrated on the CIFIST grid reduced to $-1.0 \leq [\mathrm{Fe/H}] \leq 0.5$.




(Magic+13 is the more accurate prescription)

# Final implementation