Fermi acceleration in magnetized turbulence

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Collaborators:

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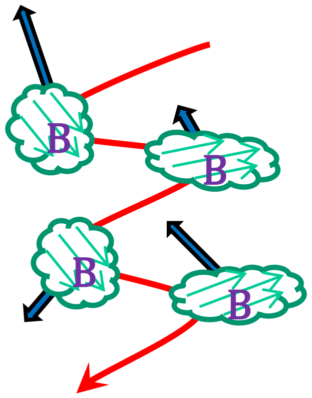
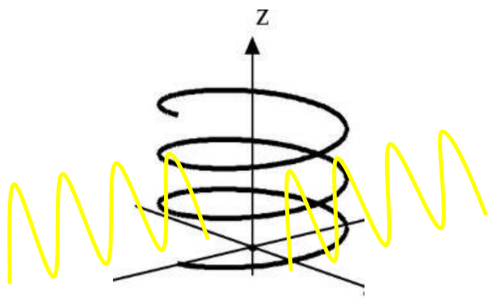
Two pictures for particle acceleration in magnetized turbulence

→ Original Fermi acceleration[1]: scattering off moving magnetic scatterers, with **E=0** in local rest frame



isotropic + elastic scattering in scattering center rest frame
$\Rightarrow \Delta p > 0$ for head-on, $\Delta p < 0$ tail-on

→ Quasilinear theory: transport in a bath of linear waves (e.g. Alfvén, magnetosonic)… energy gain through resonant interactions[2]



… interactions dominated by resonances, e.g. $k\, r_g \sim 1$

→ in phenomenology… Fokker-Planck equation: $$\frac{\partial}{\partial t} f(p,t) = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2\, D_{pp}\, \frac{\partial}{\partial p} f(p,t)\right]$$

→ issues:

1. how to calculate the diffusion coefficient D_{pp} in realistic environments + strong turbulence?
2. relativistic regime?
3. meanwhile, PIC simulations invalidate diffusive Fokker-Planck!

Refs: 1. Fermi 49, 54. 2. e.g. Kennel + Engelmann 66, …, R. Schlickeiser 02 + refs;

Implementing stochastic Fermi acceleration in a large-scale, random flow

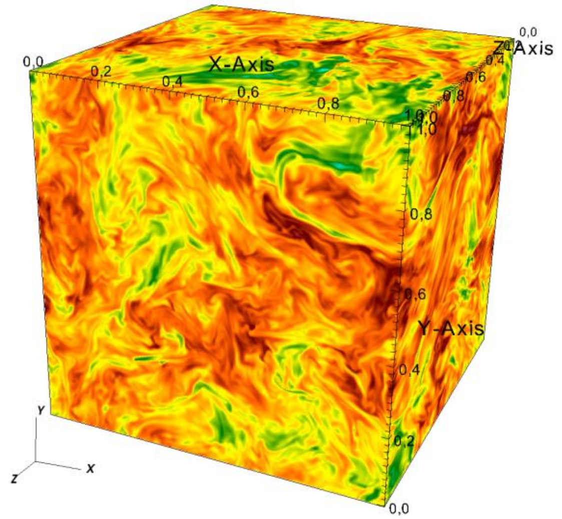
→ ***what matters is the shear of the velocity flow $\partial_\alpha {u_E}^\beta$***:

ideal MHD conditions: $\boldsymbol{E}$ vanishes in frame moving at $\boldsymbol{u_E} \propto \boldsymbol{E} \times \boldsymbol{B}$

⇒ no acceleration in absence of shear!

… in original Fermi scenario: shear ↔ difference in velocity of scattering centers

… in turbulent flow: shear $\partial_\alpha {u_E}^\beta \supset$ compression, shear, acceleration…



© C. Demidem, MHD turb.

→ ***particles probe turbulence on scales ≳ gyroradius***:

… gradients/shear $\partial_\alpha {u_E}^\beta$ are distributed on all scales…

… particles are insensitive to small-scale physics, but trapped/accelerated in large-scale structures

⇒ particles see turbulence coarse-grained on scale of gyroradius

Refs: 1. Bykov+Toptygin 81, Ptuskin 88, …, Ohira 13, ML19, Demidem+20, ML21

Generalized Fermi acceleration: follow the frame where E=0

→ convenient choice[1]: follow particle momentum p' in (accelerated!) frame moving at $\boldsymbol{u_E}$

in that frame, no electric field…

⇒ Δ energy ∝ non-inertial forces characterized by velocity shear

→ approximation[2]: $$\frac{\mathrm{d}p'}{\mathrm{d}t'} = -p'\left\{\ldots \boldsymbol{a_E}\cdot\boldsymbol{b} + \ldots\,\Theta_{\parallel} + \ldots\,\Theta_{\perp}\right\}$$

effective gravity along field line

$$\boldsymbol{a_E} = u_E^{\alpha}\,\partial_\alpha\,\boldsymbol{u_E}$$

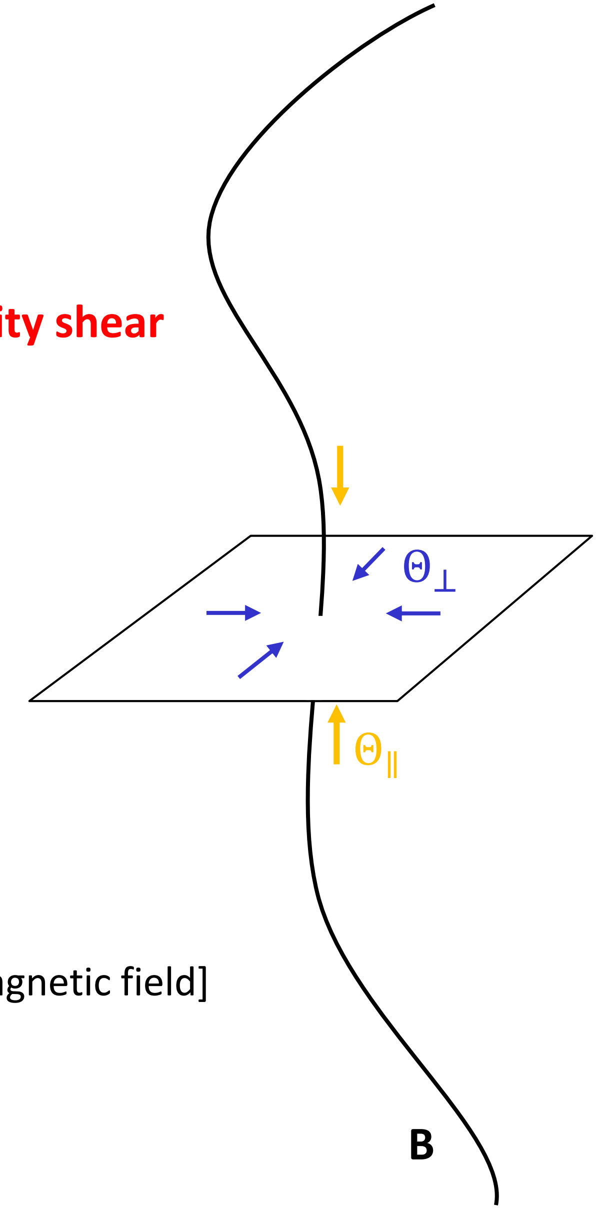
velocity shear along field line

$$\Theta_{\parallel} = b^\alpha b^\beta \partial_\alpha u_{E\,\beta}$$

compression transverse to field line

$$\Theta_{\perp} = \left(\eta^{\alpha\beta} - b^\alpha b^\beta\right)\partial_\alpha u_{E\,\beta}$$

[considers all scales ≫ r_g, ignores scales ≪ r_g , assumes local gyromotion around curved magnetic field]

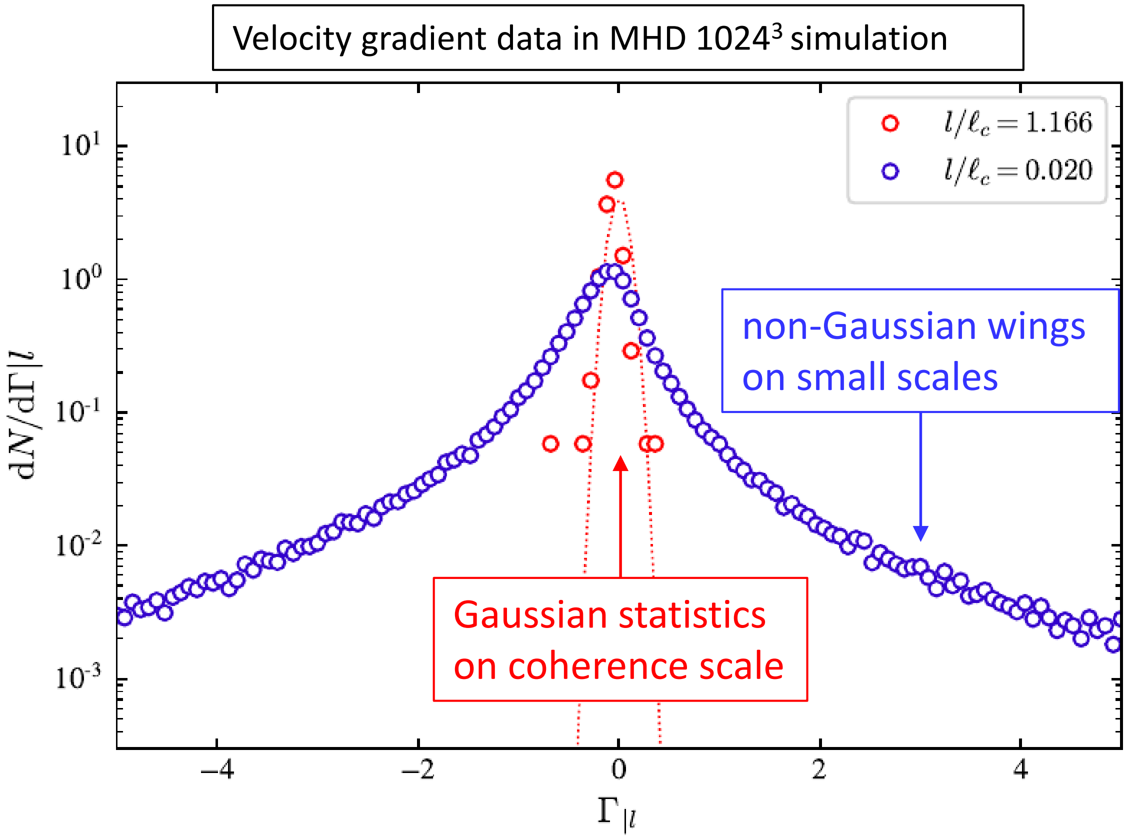


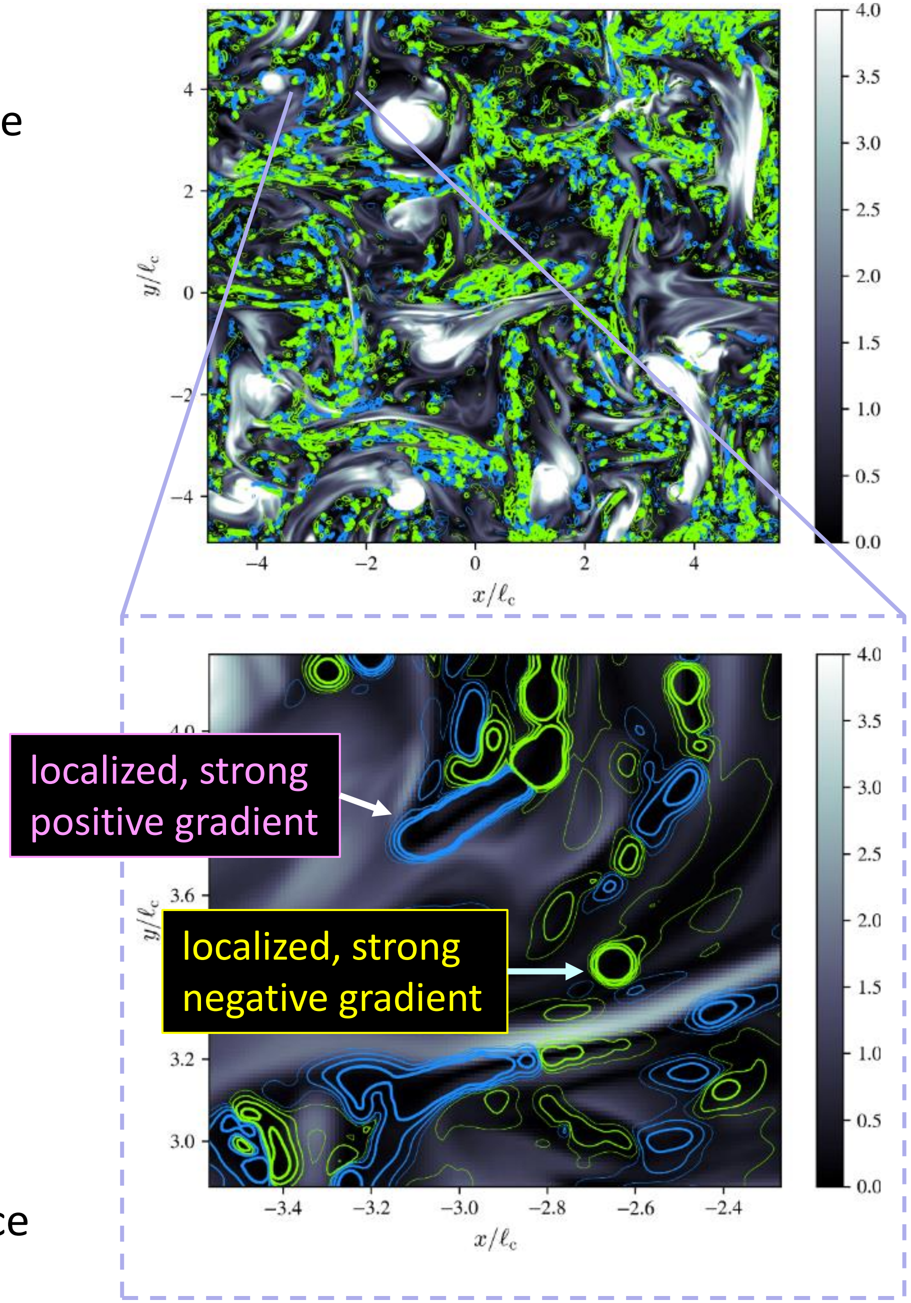
→ see talk by Virginia Bresci for comparison to PIC simulations

Refs: 1. ML19 2. ML21

Fermi acceleration in magnetized turbulence… the role of non-Gaussian gradients

→ **statistics:** velocity gradients become increasingly non-Gaussian (intermittent) at small scales (↔ small gyroradii), taking large values in localized regions…





NB: Γ_l represents velocity gradient (acceleration, $\Theta_{\parallel}$, $\Theta_{\perp}$), i.e. stochastic force acting on particle, coarse-grained on scale l ~ gyroradius

From the statistics of the random force to the transport equation

→ **two pictures:** a microscopic random walk for the evolution of one particle momentum…
… in correspondence with a kinetic equation for the distribution function *f(p,t)*…

Microscopic picture: random walk	⟷	Kinetic transport equation
$\frac{\mathrm{d}}{\mathrm{d}t} \ln p \sim \Gamma(t)$ Γ: stochastic force		$\frac{\partial}{\partial t} f(p,t) \sim \ldots$

Brownian motion: Γ gaussian white noise ⟷ Fokker-Planck: $\frac{\partial}{\partial t} f(p,t) \sim \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D_{pp} \frac{\partial}{\partial p} f(p)\right]$

Here: Γ powerlaw distributed as gradients ⟷ non-Fokker-Planck equation: powerlaw tails generic

Lévy flights: Γ heavy-tailed up to ∞ ⟷ Fractional Fokker-Planck: e.g. see L. Vlahos

Fermi acceleration in magnetized turbulence… from first principles

→ scheme: capture (non-Gaussian) statistics of velocity gradients using a multifractal description of turbulence[1]
+ formulate kinetic equation (non-Fokker-Planck)[2]…

→ details: scaling… $\Gamma_l \sim \Gamma_{\ell_c}\ (l/\ell_c)^h$

gradient on coherence scale ℓ_c:
Gaussian distributed, $\Gamma_{\ell_c} \sim \langle \delta u^2 \rangle^{1/2}/\ell_c$

scaling index h: random variable characterized by prob. law

distribution… $\mathrm{Prob.}\,(\Gamma_l) \sim \mathrm{Prob.}\,(\Gamma_{\ell_c}) \otimes \mathrm{Prob.}\,(h)$

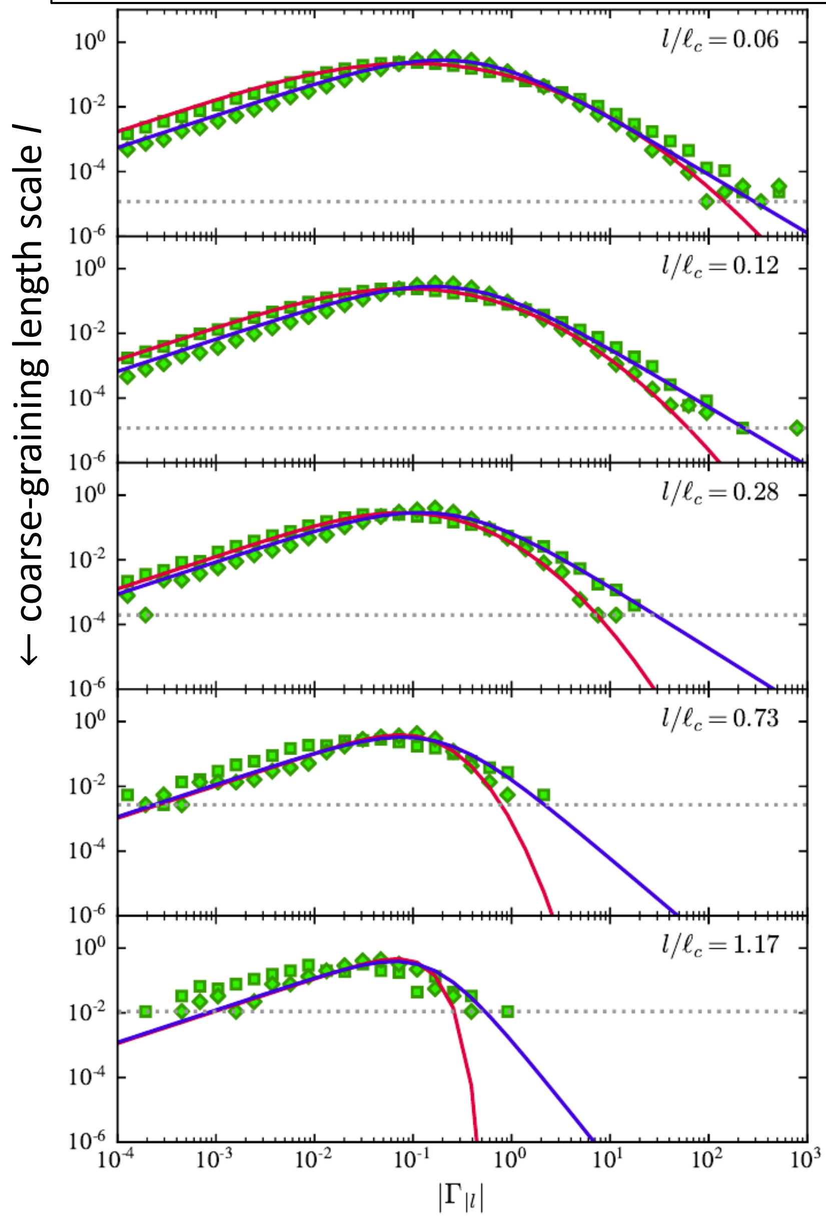
momentum jumps: $\Delta \ln p \sim \Gamma_l\ \Delta t \ \Rightarrow\ \mathrm{Prob.}\,(\Delta \ln p) \sim \mathrm{Prob.}\,(\Gamma_l)$

→ kinetic equation…

$$\partial_t\, n_p = \int_0^{+\infty} \mathrm{d}p' \left[\frac{\varphi\,(p|p')}{t_{p'}} n_{p'}(t) - \frac{\varphi\,(p'|p)}{t_p} n_p(t) \right]$$

$$n_p = \frac{\mathrm{d}N}{\mathrm{d}p} \qquad t_p \sim r_{\rm g}/c$$

Velocity gradient data in MHD 1024^3 sim. + model

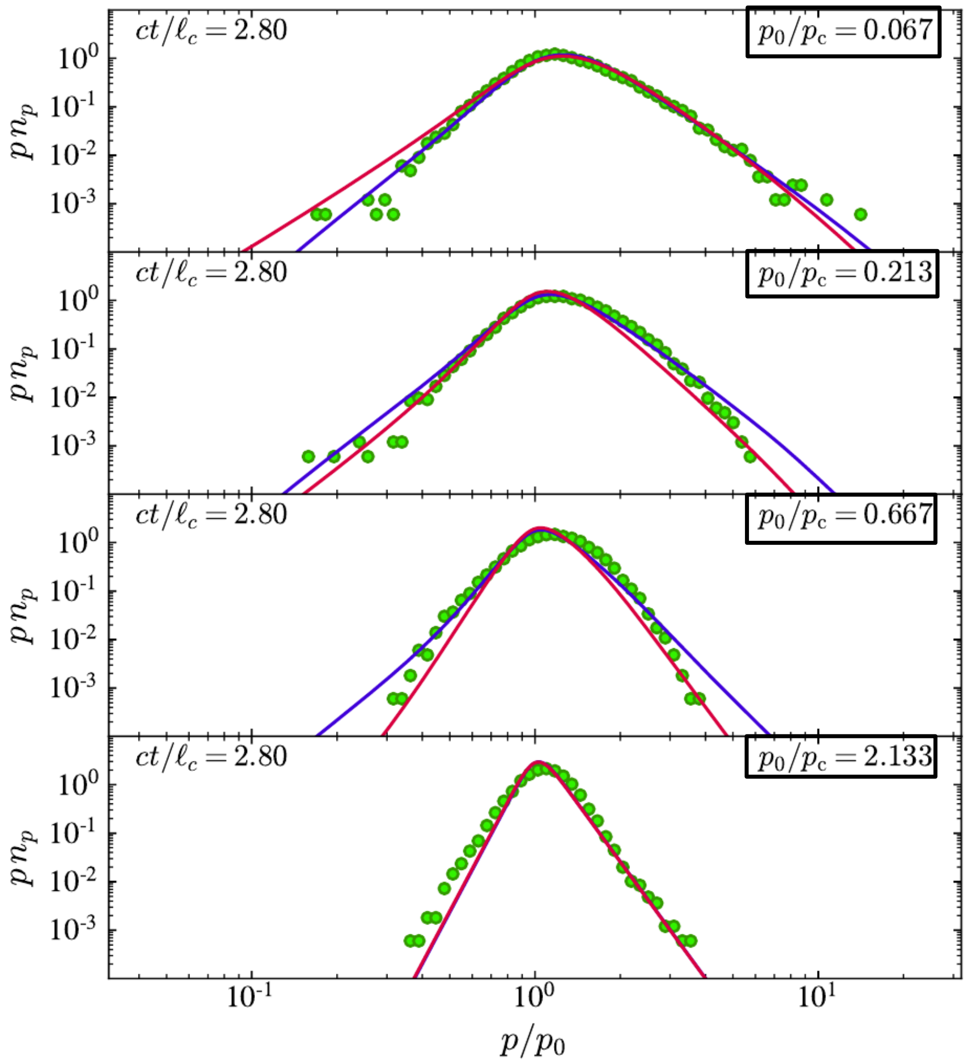


Refs.: 1. Parisi+Frisch 85 2. ML 22 in prep

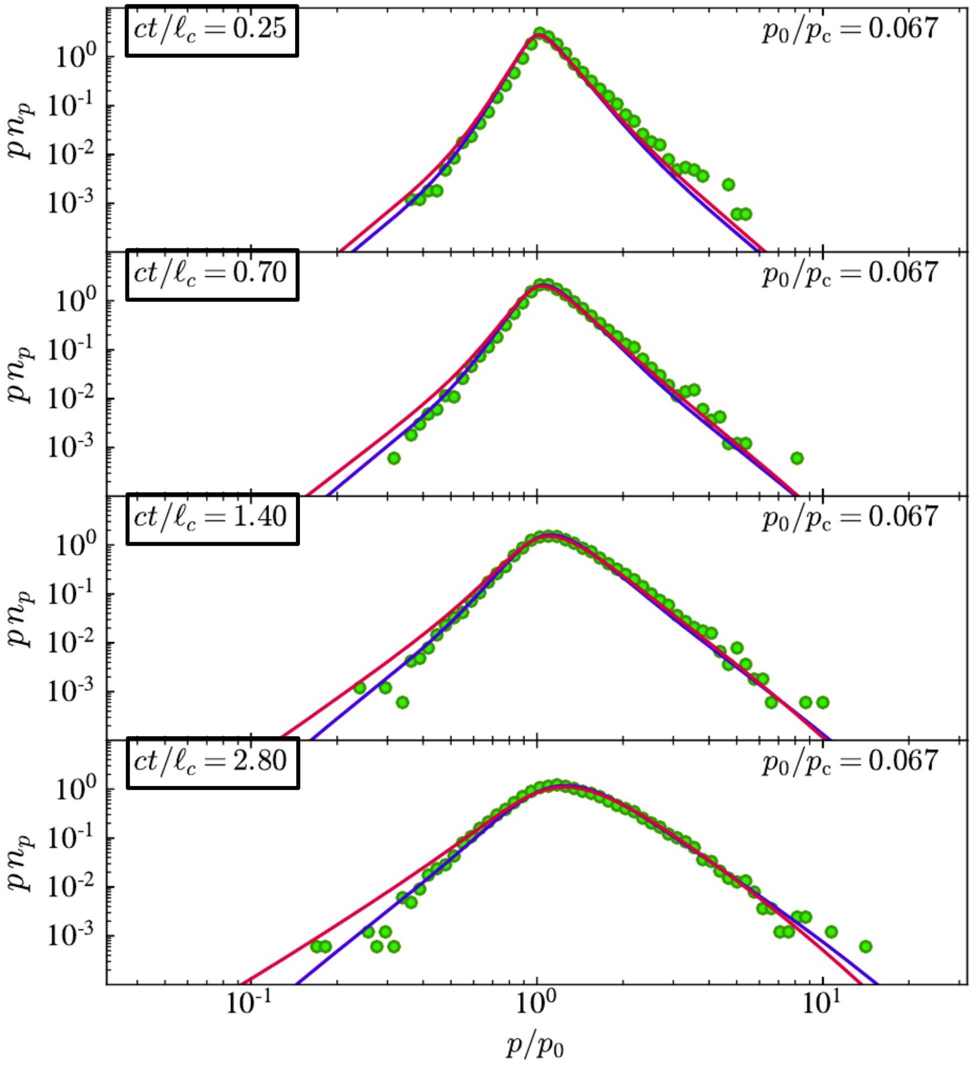
Fermi acceleration in magnetized turbulence… from first principles

→ **comparison to numerical data:** integrate kinetic equation and compare solution (Green function) to distribution measured in MHD 1024^3 simulation by time-dependent particle tracking…

← increasing initial particle momentum p_0



← increasing integration time

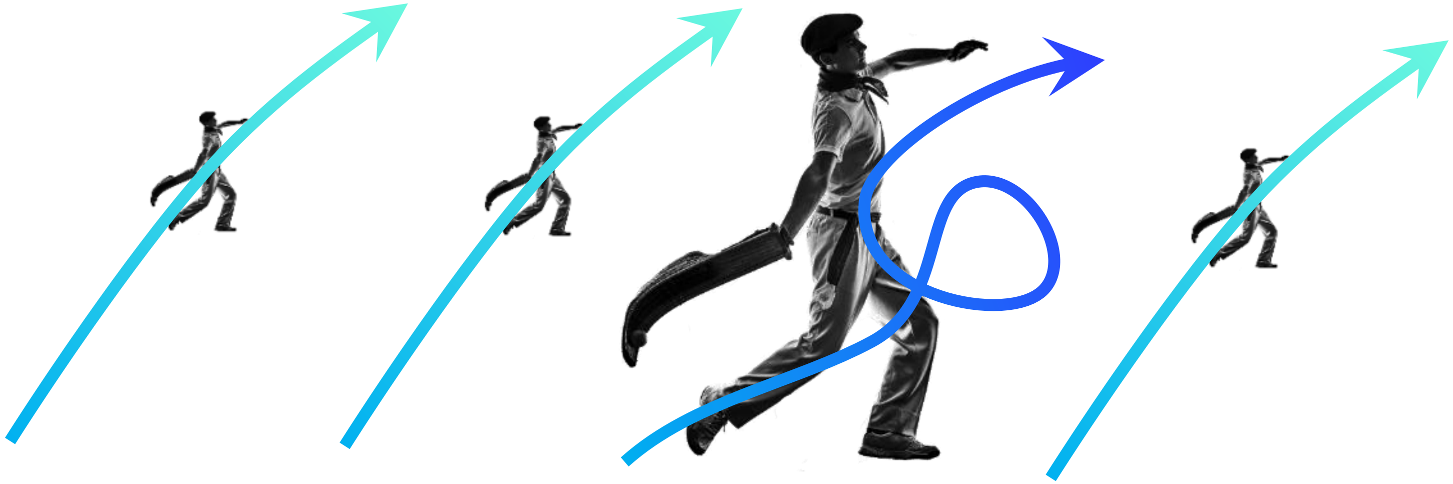


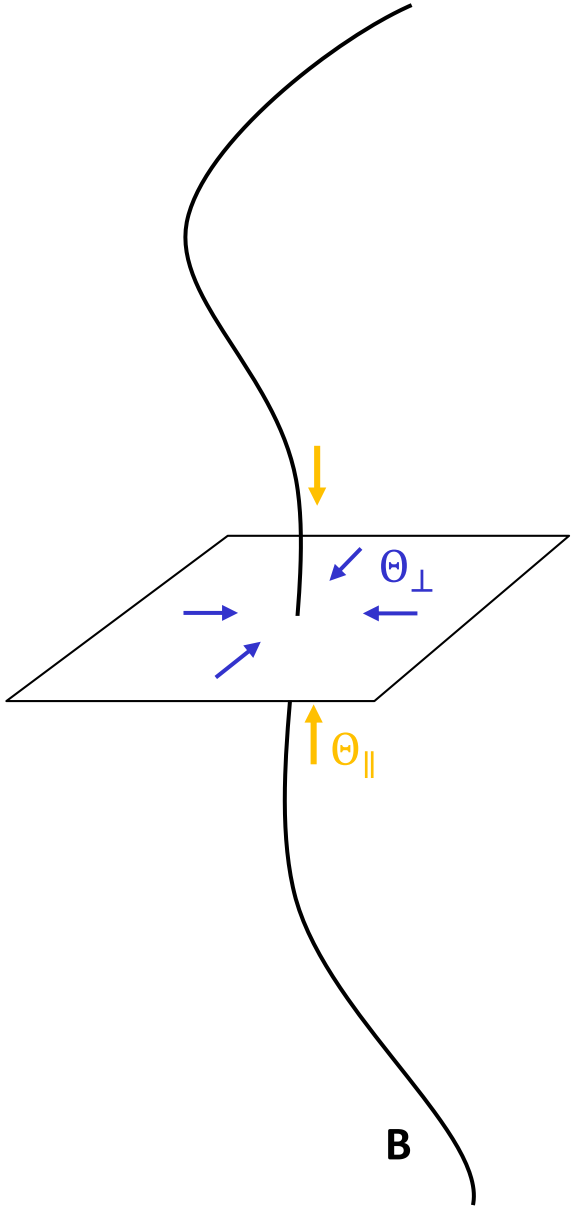
⇒ transport equation can reproduce time- and energy- dependent Green functions… + explain origin of powerlaw spectra

Fermi acceleration in one sketch

→ the traditional view: stochastic acceleration as *Brownian motion* (→ Fokker-Planck)

→ the present view: stochastic interactions with *intermittent gradients* (→ powerlaw spectra)





Outline:

→ Implementing Fermi acceleration in a realistic turbulence setting:

*… track particle history in frame in which **E=0**…*
… particles are accelerated in regions of strong velocity gradients!

→ Derive a kinetic transport equation for Fermi acceleration:

… velocity gradients are non-Gaussian on small scales: failure of Fokker-Planck description…
… velocity gradients statistics can be captured through multi-fractal model of turbulence…
… kinetic equation provides fair reconstruction of time+energy dependent Green functions…